ALBOURNE C.E. PRIMARY SCHOOL

'On our learning journey together'.

Albourne CE Primary School

Progression in Calculation

Adapted from Babcock 2015

Aims

The national curriculum for mathematics aims to ensure that all pupils:

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Introduction

Written methods of calculations are based on mental strategies. Each of the four operations builds on mental skills which provide the foundation for jottings and informal written methods of recording. Skills need to be taught, practised and reviewed constantly. These skills lead on to more formal written methods of calculation.

Strategies for calculation need to be represented by models and images to support, develop and secure understanding. This, in turn, builds fluency. When teaching a new strategy it is important to start with numbers that the child can easily manipulate so that they can understand the methodology.

The transition between stages should not be hurried as not all children will be ready to move on to the next stage at the same time, therefore the progression in this document is outlined in stages. Previous stages may need to be revisited to consolidate understanding when introducing a new strategy.

A sound understanding of the number system is essential for children to carry out calculations efficiently and accurately.

Magnitude of Calculations

Year 1 – U + U, U + TU (numbers up to 20) including adding zero, U – U, TU – U (numbers up to 20) including subtracting zero, U x U, U ÷ U

Year 2 - TU + U, TU + multiples of 10, TU + TU, U + U + U, TU - U, TU – tens, TU – TU, TU x U, U ÷ U

Year 3 – add numbers with up to three-digits, HTU + multiples of 10, HTU + multiples of 100, subtract numbers up to three-digits, HTU – U, HTU – multiples of 10, HTU – multiples of 100, HTU – HTU, TU x U, TU ÷ U

Year 4 - add and subtract numbers with up to four-digits, ThHTU + ThHTU, ThHTU - ThHTU, add and subtract decimals with up to two decimal places in the context of money, multiply three numbers together, TU x U, HTU x U, TU x U, multiply by zero and one, TU ÷ U, HTU ÷ U

Year 5 – add and subtract numbers with more than four-digits, add and subtract decimals with up to three decimal places, ThHTU x U, ThHTU x TU, HTU x TU, multiply whole numbers and decimals with up to three-decimal places by 10, 100 and 1000, divide numbers with up to four-digits by U (including remainders as fractions and decimals and rounding according to the context)

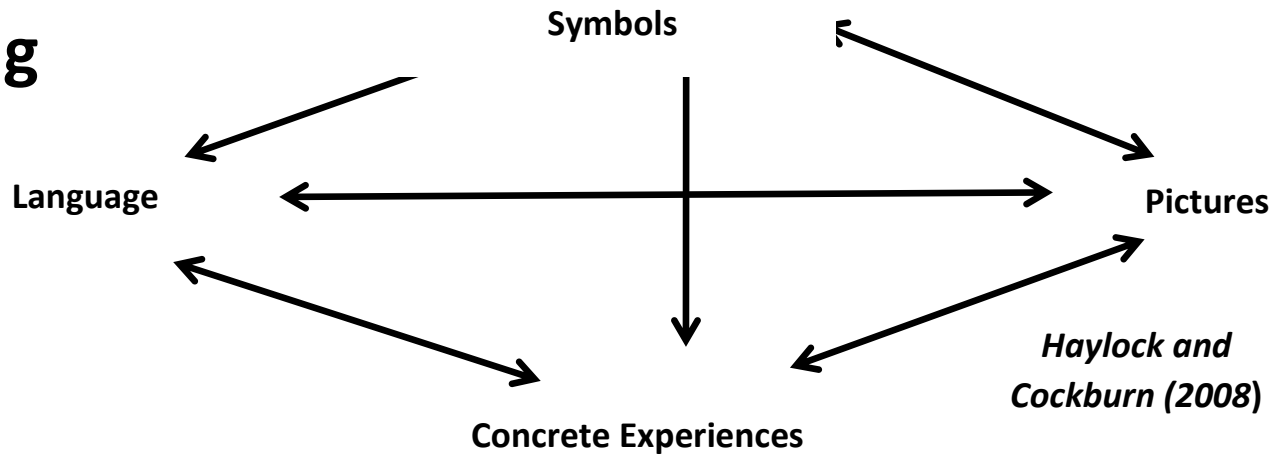
Year 6 - add and subtract numbers with more than four-digits, add and subtract decimals with up to three decimal places, multiply numbers with up to four-digits by TU, multiply numbers with up to two-decimal places by a whole number, divide numbers up to four-digits by TU (interpreting remainder according to the context), divide decimals up to two-decimal places by U or TU

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. … pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge to science and other subjects.

National Curriculum 2014

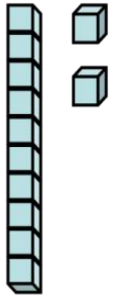
Structuring Learning

Children must have concrete experiences that enable them to create visual images. They should be encouraged to articulate their learning and to become pattern spotters.

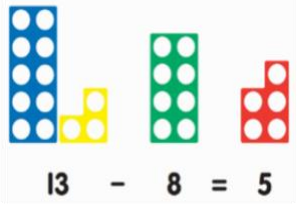


Haylock and Cockburn (2008)

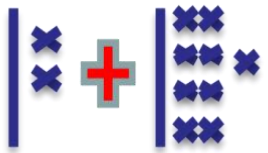
Active/concrete



Communicating Mathematically



Building visual images



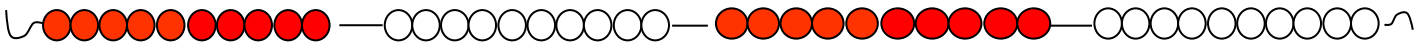
Pattern Spotting

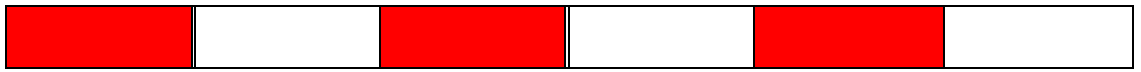
13 - 8

Abstract

12 + 19

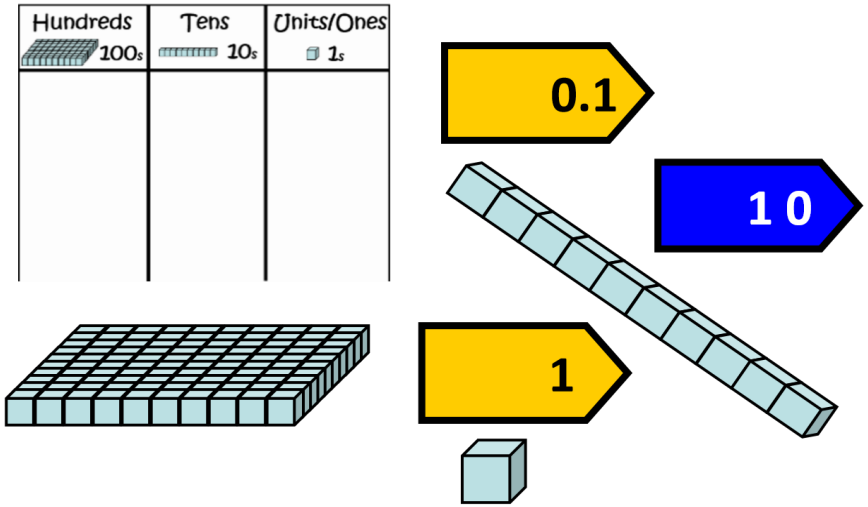
bead string





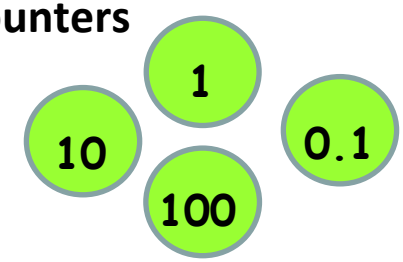
counting stick

place value apparatus



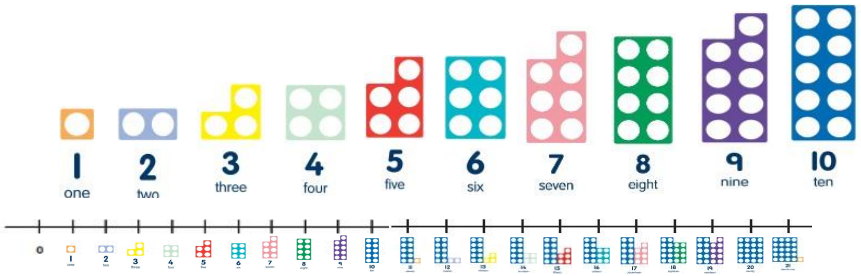
Multilink

place value counters

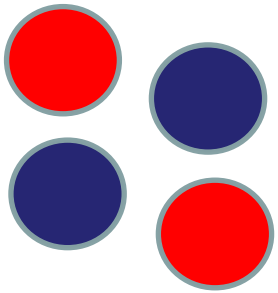


Cuisenaire

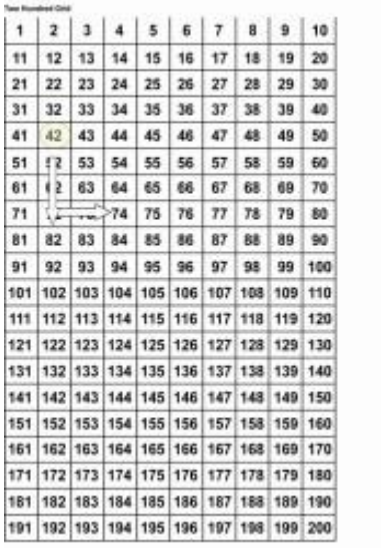
Numicon

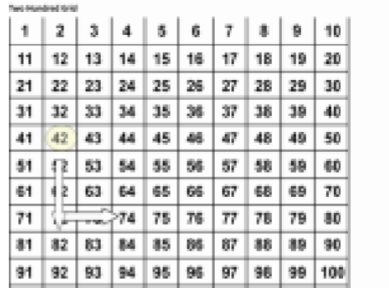


number line



doubled-sided counters





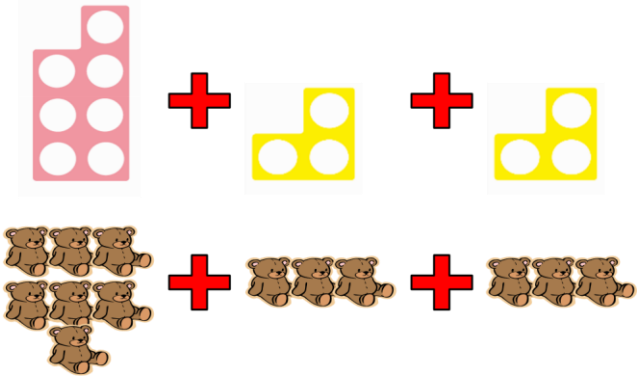
number grids 100 and 200

Structures of Addition (Haylock and Cockburn 2008)

Children should experience problems with all the different addition structures in a range of practical and relevant contexts e.g. money and measurement.

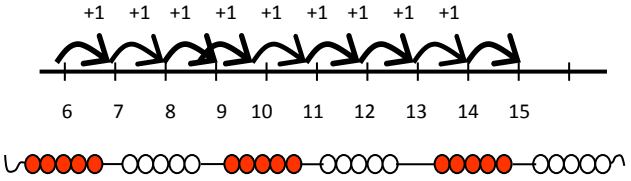
Aggregation

Union of two sets
How many/much altogether?
The total



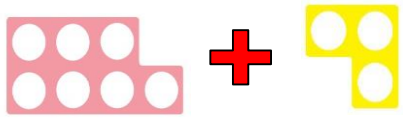
Augmentation

Start at and count on
Increase by
Go up by

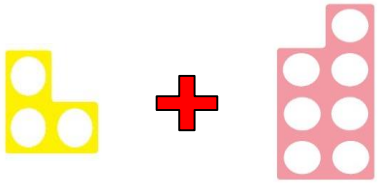


Commutative law

Understand addition can be done in any order.
Start with bigger number when counting on
(Explain to children that subtraction does not have this property)



is the same as/equal to (=)



Addition	Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.

End of Year Expectations

Reception

Children say which number is one more than a given number

Using quantities and objects, they add two single-digit numbers and count on to find the answer

Year 1

Read, write and interpret mathematical statements involving addition and equals signs

Represent and use number bonds within 20

Add one-digit and two-digit numbers to 20, including zero

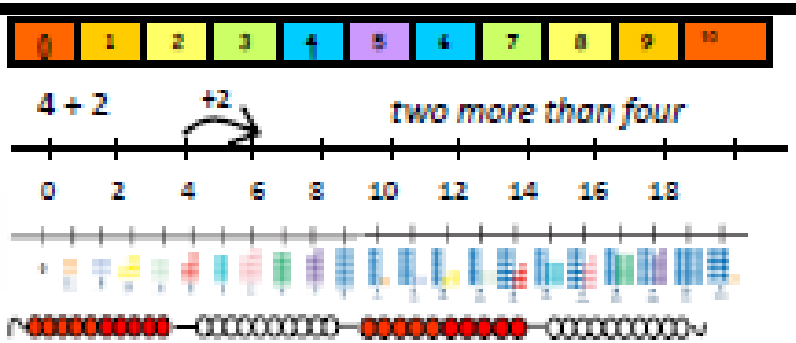
Solve one-step problems that involve addition using concrete objects and pictorial representations, and missing number problems such as 7=X-9

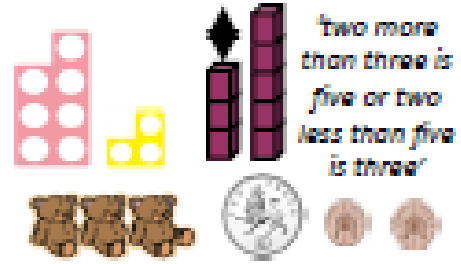
Count to and across 100, forwards and backwards, from any number including 0

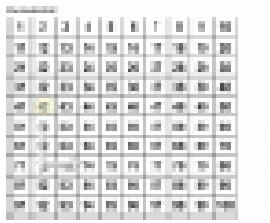
Given a number identify one more

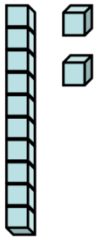
Count in multiples of 2s, 5s and 10s

Possible Concrete and Visual Representations

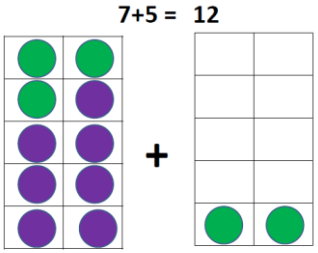






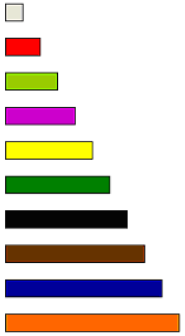


Ten frames



I had 7, then I counted on 5...

Cuisenaire

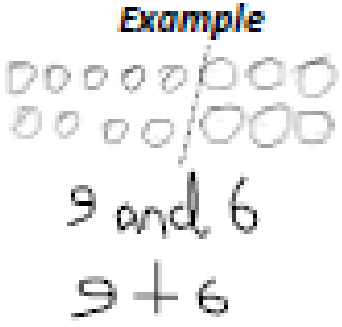


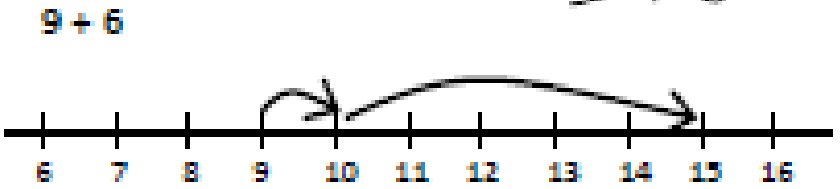
Use practical resources such as bears, counters, cubes and number lines/hundred grids and progress to a resource such as Numicon to encourage counting in groups rather than ones

Children's recording

If using Numicon, children could use printed Numicon icons and stick these in - progressing to recording number sentences alongside

Children may record pictorially progressing to recording number sentences alongside





Addition	Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.

End of Year Expectations

Year 2

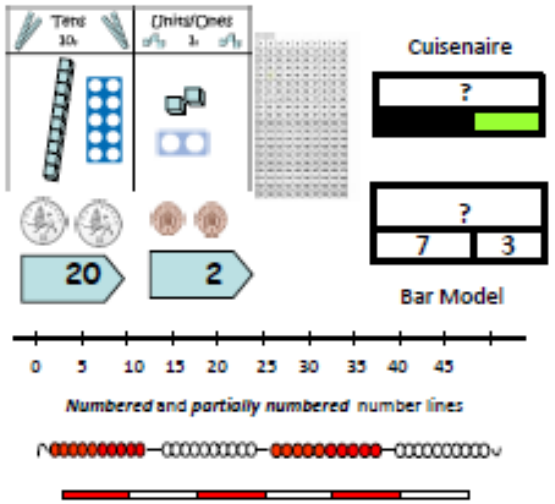
Solve problems with addition: •using concrete objects and pictorial representations, including those involving numbers, quantities and measures. •applying their increasing knowledge of mental and written methods

Show that addition of two numbers can be done in any order (commutative).(e.g. 5 + 2 + 1 = 1 + 5 + 2 = 1 + 2 + 5)

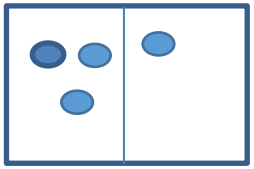
Recall and use addition facts to 20 fluently, and derive and use related facts up to 100.

Add numbers using concrete objects, pictorial representations and mentally, including: •a two digit number and ones, •a two digit number and tens, •two 2 digit numbers, •adding three one digit numbers

Possible Concrete and Visual Representations



Use Numicon, number grids, place value apparatus/Dienes, place value grids, place value cards. Encourage children to partition numbers rather than counting in ones.



Counters. Show that if you swap them over the total is still the same.

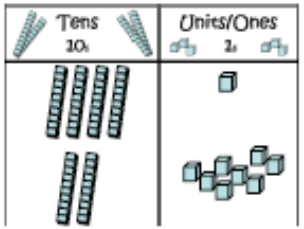
Inverse and missing number slides 10 – 3 = ?

7 + ? = 10

Teacher Modelling/Children's Recording

Children apply, develop and secure their understanding of place value.

Use jottings and record number sentences

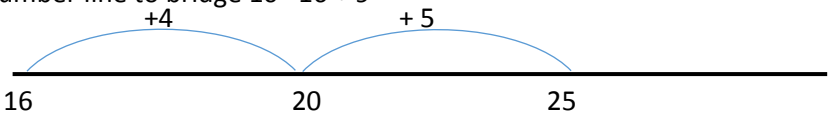


41
+
28

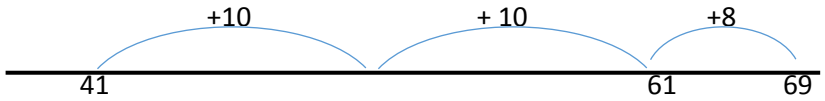
40	1	
+ 20	+ 8	
= 60	= 9	60 + 9 = 69

41 + 28 40 +20 = 60 1+8 = 9 60+9 =69

number line to bridge 10 16 + 9



number line to add on 41 +28



Addition	Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.

End of Year Expectations

Year 3

Add and subtract numbers mentally, including: a three-digit number and ones, a three-digit number and tens, a three-digit number and hundreds

Add numbers with up to three digits, using formal written methods of columnar addition

Estimate the answer to a calculation and use inverse operations to check answers

Solve problems, including missing number problems, using number facts, place value, and more complex addition

Count from 0 in multiples of 4, 8, 50 and 100;

Find 10 or 100 more than a given number

Year 4

Add numbers with up to 4 digits using the formal written methods of columnar addition where appropriate

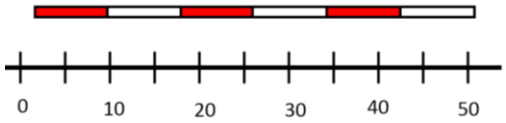
Estimate and use inverse operations to check answers to a calculation

Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why

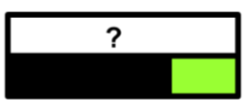
Count in multiples of 6, 7, 9, 25 and 1000

Find 1000 more or less than a given number

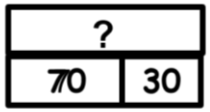
Possible Concrete and Visual Representations



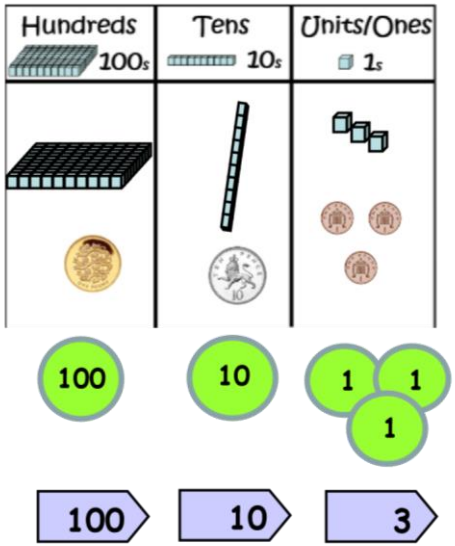
Partially numbered **and** blank number lines



Cuisenaire



Bar Model



Addition	**Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.**

Teacher Modelling/Children's Recording

Children apply, develop and secure their understanding of place value and begin to record in columns

Manipulatives SHOULD be used alongside alogortihms. Breakdown each stage as below but move through rapidly.

1. Column addition (no regrouping) with up to three digits (Yr 3) and up to four digits (Yr 4)

T	U		H	T	U
40	+ 1		100	+ 40	+ 1
+ 20	+ 8		+ 100	+ 20	+ 8
60	+ 9 = 69		200	+ 60	+ 9 = 269

2. Compact (column) recording no regrouping

H	T		H	T	U
4	1		1	4	1
+ 2	8		+ 1	2	8
6	9		2	6	9

When talking through the calculations, teachers and children should use terminology of place value relating to that column e.g. 1 + 8 = 9, 40 + 20 + 10 = 70, 100 + 100 = 200.

3. Expanded recording with regrouping

T	U		H	T	U
40	+ 3		100	+ 40	+ 3
+ 20	+ 8		+ 100	+ 20	+ 8
70	+ 1 = 71		200	+ 70	+ 1 = 271
~~10~~				~~10~~	

4. Compact (column) with regrouping

H	T	U		Th	H	T	U
1	4	3		1	7	8	9
+ 1	2	8			4	0	9
2	7	1		+		2	1
~~1~~				2	2	1	9
				~~1~~	~~1~~	~~1~~	

Ensure children have the opportunity to add MORE THAN two numbers with differeing numbers of digits

£	7.89
+ £	6.42
£	14.31
~~1~~	~~1~~

Add decimals in the context of money

Addition	Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.

End of Year Expectations

Year 5

- Count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000
- Add numbers mentally with increasingly large numbers
- Add whole numbers with more than 4 digits, including using formal written methods (columnar addition)
- Using rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- Solve addition multi-step problems in contexts, deciding which operations and methods to use and why

Year 6

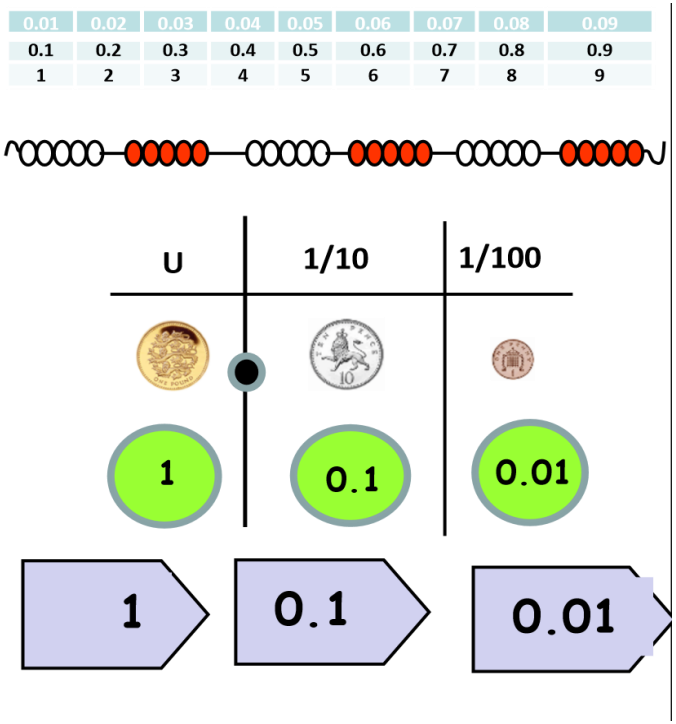
Perform mental calculations, including with mixed operations and large numbers

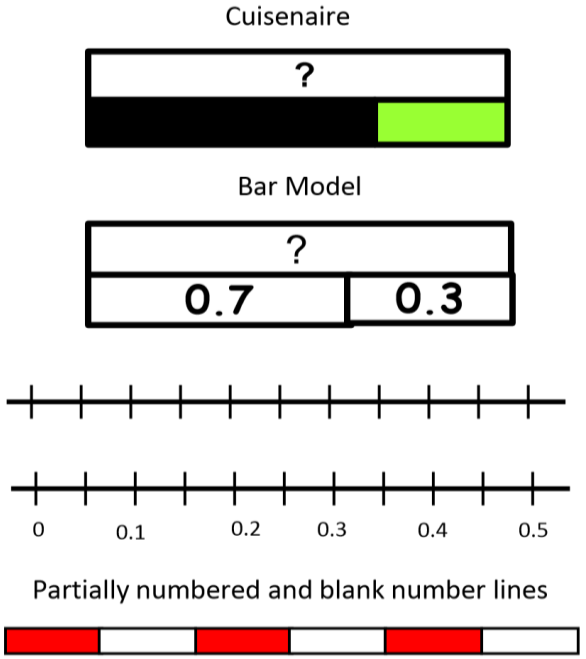
Use their knowledge of the order of operations to carry out calculations involving the four operations

Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

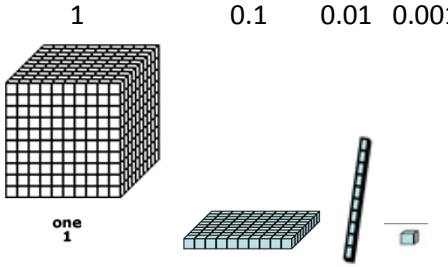
Solve problems involving addition, subtraction, multiplication and division

Possible Concrete and Visual Representations





Dienes representing



Numicon base board representing 1 and 1s as 0.01

Addition	**Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.**

Teacher Modelling/Children's Recording

Manipulatives could be used alongside algorithms

Column addition (with regrouping)

```
  ThHTU
  5 1 8 9
+ 3 1 2 8
  8 3 1 7
    1 1
```

Addition with decimals up to three decimal places including in different contexts e.g. money and measures

```
  T U  1/10 1/100 1/1000
  5 1. 8  9
    3. 1  2   8
+   0. 3
  5 4. 3  1   8
    1  1
```

Ensure children have the opportunity to add more than two numbers including decimals with differing numbers of digits

Structures of Subtraction (Haylock and Cockburn 2008)

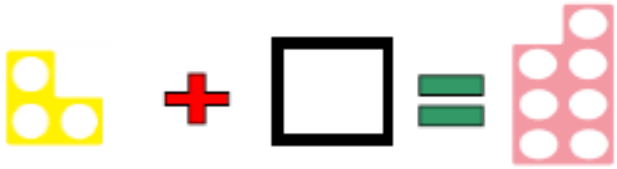
Children should experience problems with all the different subtraction structures in a range of practical and relevant contexts e.g. money and measurement

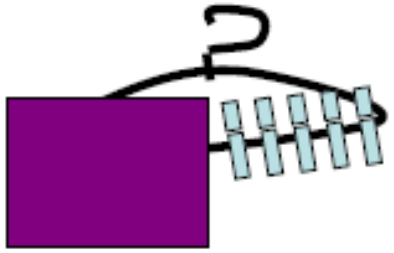
Partitioning

Take away
... how many left?
How many are not?
How many do not?

Inverse-of-addition

What must be added?
How many (much) more needed?

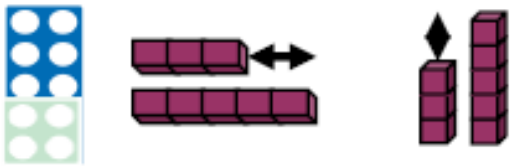




There are ten pegs on the hanger – how many are covered?

Comparison

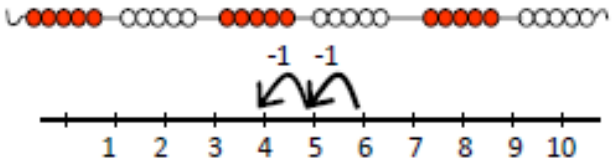
What is the difference?
How many more?
How many less (fewer)?
How How much greater?
How much smaller?



'two more than three is five or two less than five is three'

Reduction

Start at and reduce by
Count back by
Go down by



Subtraction	Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly. Addition and subtraction should be taught together.

End of Year Expectations

Reception

They say which number is one less than a given number.

Using quantities and objects, they subtract two single-digit numbers and count on or back to find the answer.

Year 1

Given a number identify one less

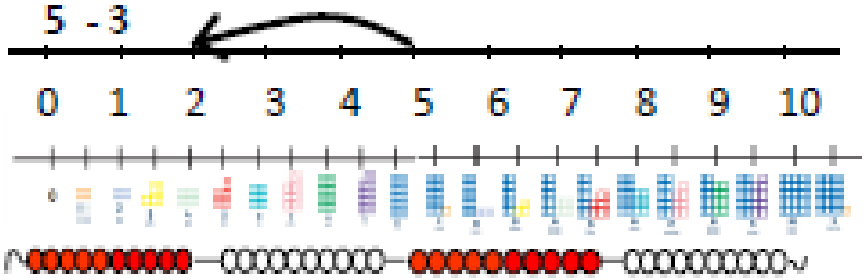
Read, write and interpret mathematical statements involving subtraction and equals signs

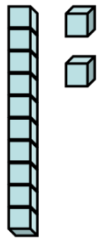
Represent and use number bonds and related subtraction facts within 20

Subtract one-digit and two-digit numbers to 20, including zero

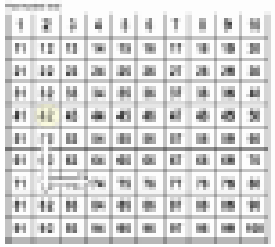
Solve one-step problems that involve subtraction, using concrete objects and pictorial representations, and missing number problems such as 7=X-9

Possible Concrete and Visual Representations

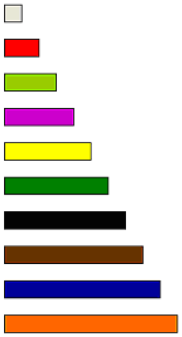




'two less than five is three'



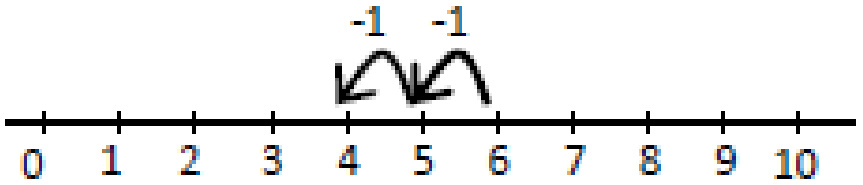
Cuisenaire



Use practical resources such as bears, counters, cubes and number lines/hundred grids and progress to a resource such as Numicon to encourage counting back in groups rather than ones

Children's Recording

Children may begin recording pictorially progressing to recording number sentences alongside



Children could use printed Numicon icons and stick these in, again progressing to recording number sentences alongside

Subtraction

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly. Addition and subtraction should be taught together.

End of Year Expectations

Year 2

Solve problems subtraction: using concrete objects and pictorial representations, including those involving numbers, quantities and measures and applying their increasing knowledge of mental and written methods

Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100.

Subtract numbers using concrete objects, pictorial representations and mentally, including: a two digit number and ones, a two digit number and tens, two 2 digit numbers

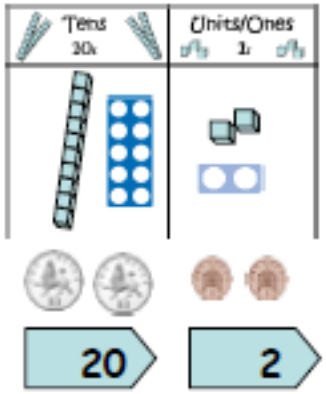
Show that subtraction of one number from another cannot be done in any order.(commutative)

Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

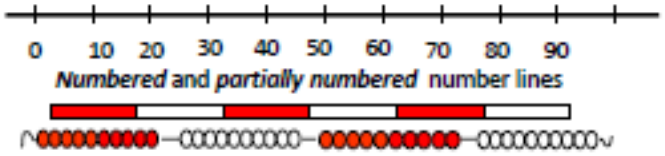
Possible Concrete and Visual Representations

10 - 4

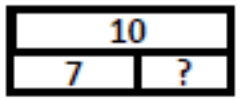
Finding the difference



Children should use concrete materials and pictorial representations, and use numbers in different contexts e,g, money and measures, Encourage children to partition numbers rather than counting in ones.

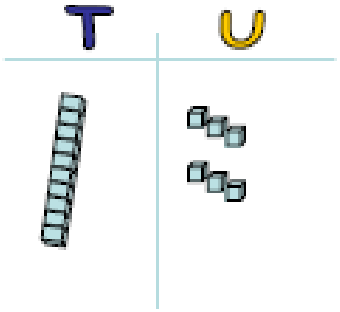


Cuisenaire Bar Model



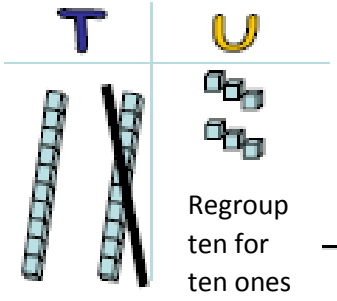
Children's Recording

Children apply, develop and secure their understanding of place value and begin to record using jottings, numberlines and number sentences

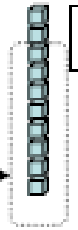


16 - 3

no regrouping

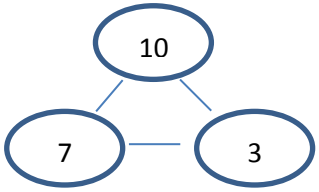


Regroup ten for ten ones



26 - 8

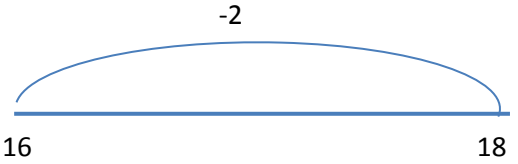
regrouping



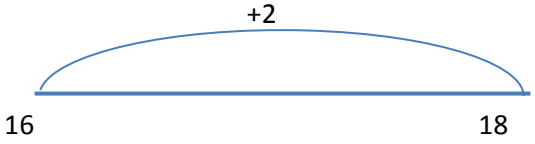
7 + 3 = 10
3 + 7 = 10
10 - 7 = 3
10 - 3 = 7

Use numberlines to count on or count back depending on numbers e.g.

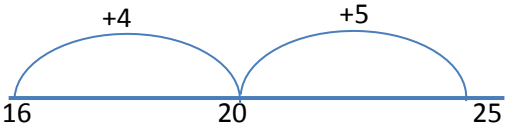
18-2=16 count back



18-16 = 2 count on



25-16 = 9



Subtraction	Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly. Addition and subtraction should be taught together.

End of Year Expectations

Year 3

Find 10 or 100 less than a given number

Subtract numbers mentally, including: a three-digit number and ones, a three-digit number and tens, a three-digit number and hundreds

Subtract numbers with up to three digits, using formal written methods of columnar subtraction

Estimate the answer to a calculation and use inverse operations to check answers

Solve problems, including missing number problems, using number facts, place value, and more complex subtraction

Year 4

Find 1000 more or less than a given number

Subtract numbers with up to 4 digits using the formal written methods of columnar subtraction where appropriate

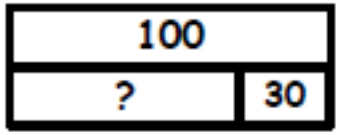
Estimate and use inverse operations to check answers to a calculation

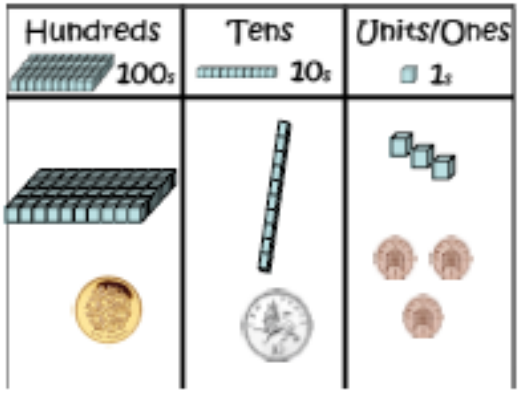
Solve subtraction two-step problems in contexts, deciding which operations and methods to use and why

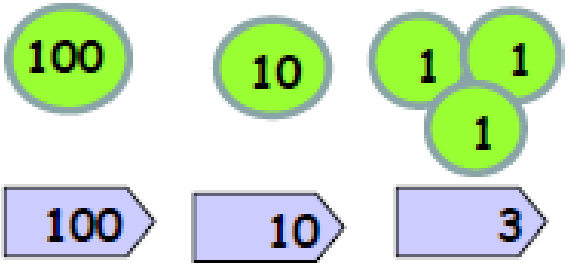
Possible Concrete and Visual Representations

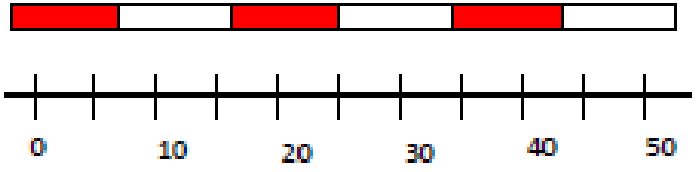
Cuisenaire

Bar Model









Subtraction	**Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly. Addition and subtraction should be taught together.**

Teacher Modelling/Children's Recording

1. Expanded column addition (no regrouping) with up to three digits (Yr 3) and up to four digits (Yr 4)

68-23

T U
60 + 8
- 20 + 3
40 + 5 = 45

148-121

H T U
100 + 40 + 8
- 100 + 20 + 1
0 + 20 + 7 = 27

When talking through the calculations, teachers and children should use terminology of place value relating to that column e.g. 1 + 8 = 9, 40 + 20 + 10 = 70, 100 + 100 = 200.

2. Compact (column) recording no regrouping

H T
68
- 23
45

HTU
148
- 121
27

3. Expanded recording with regrouping

63-28

T U
^{50}~~60~~ + $^{10+}$3
- 20 + 8
30 + 5 = 35

243-128

H T U
200 + ^{30}~~40~~ + $^{10+}$3
- 100 + 20 + 8
100 + 10 + 5 = 115

4. Compact (column) with regrouping

723-317

H T U
7 ^{1}~~2~~ ^{1}3
- 3 1 7
4 0 6

732-367

H T U
^{6}~~7~~ ^{11}~~2~~ ^{1}3
- 3 6 7
3 5 6

£^{6}~~7~~.^{11}~~2~~ ^{1}3
£3.67
£3.56

Subtract decimals in the context of money

Ensure children can solve caluclations where zero is the place holder

Subtraction	Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly. Addition and subtraction should be taught together.

End of Year Expectations

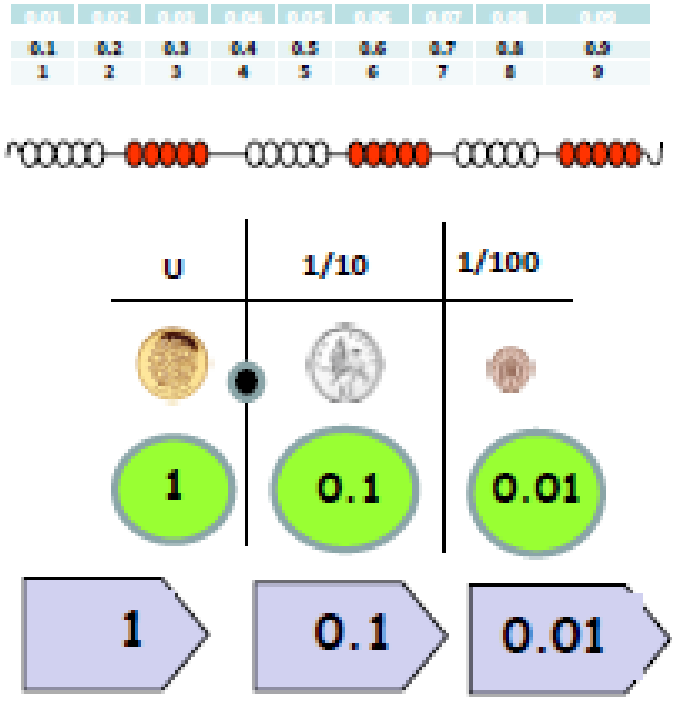
Year 5

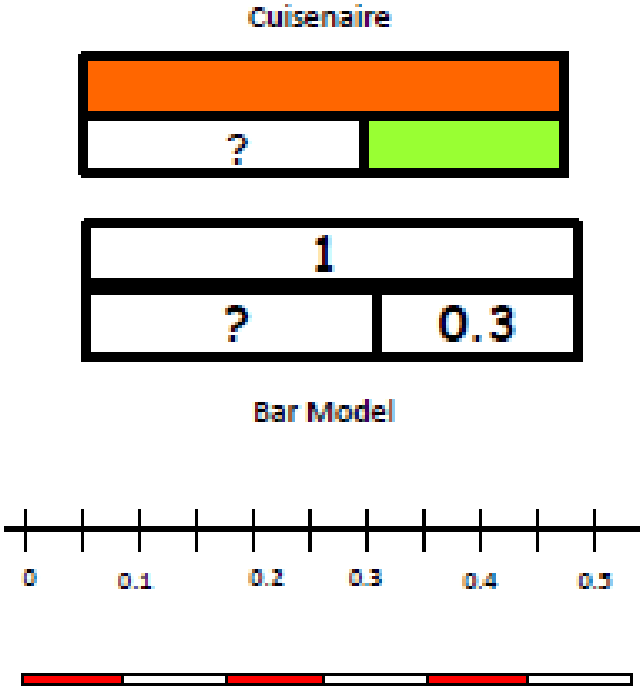
- Subtract numbers mentally with increasingly large numbers
- Subtract whole numbers with more than 4 digits, including using formal written methods (columnar subtraction)
- Using rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

Year 6

- Use their knowledge of the order of operations to carry out calculations involving the four operations
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- Solve problems involving addition, subtraction, multiplication and division
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

Possible Concrete and Visual Representations





Subtraction	Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly. Addition and subtraction should be taught together.

Teacher Modelling/Children's Recording

Manipulatives could be used alongside algorithms

1. Column subtraction (no regrouping)

Tth	Th	H	T	U
	1	3	5	4	8
-	1	2	1	2	8
		1	4	2	0

When talking through the calculations, teachers and children should use terminology of place value relating to that column e.g. 1 + 8 = 9, 40 + 20 + 10 = 70, 100 + 100 = 200.

2. Column subtraction (with regrouping)

Tth	Th	H	T	U
1	$^{2}\not{3}$	$^{13}\not{4}$	$^{11}\not{2}$	$^{1}3$
- 1	2	6	7	8
		7	4	5

Ensure children can solve calculations where zero is a place holder

Subtraction with decimals up to three decimal places including in different contexts e.g. money and measures

3. Column subtraction (no regrouping)

T	U	$\frac{1}{10}$	$\frac{1}{100}$
1	.	8	9
- 1	.	2	1
0	.	2	7

Ensure children have the opportunity to add more than two numbers

4. Column subtraction (with regrouping)

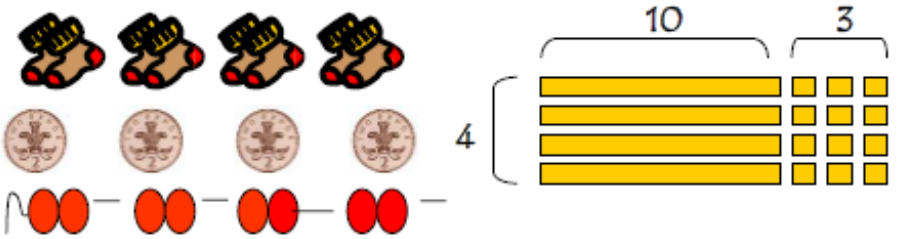
T	U	$\frac{1}{10}$	$\frac{1}{100}$
$^{6}\not{7}$	.	$^{11}\not{2}$	$^{1}3$
1	.	2	1
3	.	5	6

Structures of Multiplication (Haylock and Cockburn 2008)

Children should experience problems with all the different multiplication structures in a range of practical and relevant contexts e.g. money and measurement

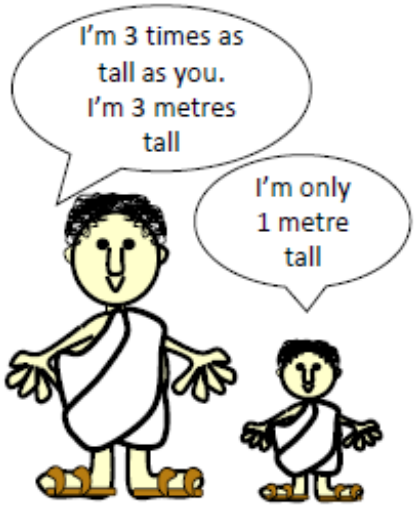
Repeated addition

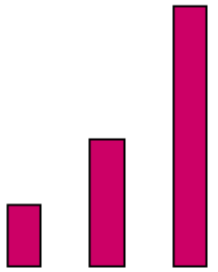
So many lots (sets) of so many
How many (how much) altogether
Per, each



Scaling

Scaling, scale factor
Doubling, trebling
So many times bigger than (longer than, heavier than, and so on)
So many times as much as (or as many as)





scaling with Cuisenaire

Commutative law

Understand multiplication can be done in any order
(Explain to children that division does not have this property)

a x b and b x a are equal

4 x 2 is the same as/equal to 2 x 4

Multiplication	**Pupils develop the concept of multiplication and division and are enabled to use these operations flexibly. Multiplication and division should be taught together.**

End of Year Expectations

Reception

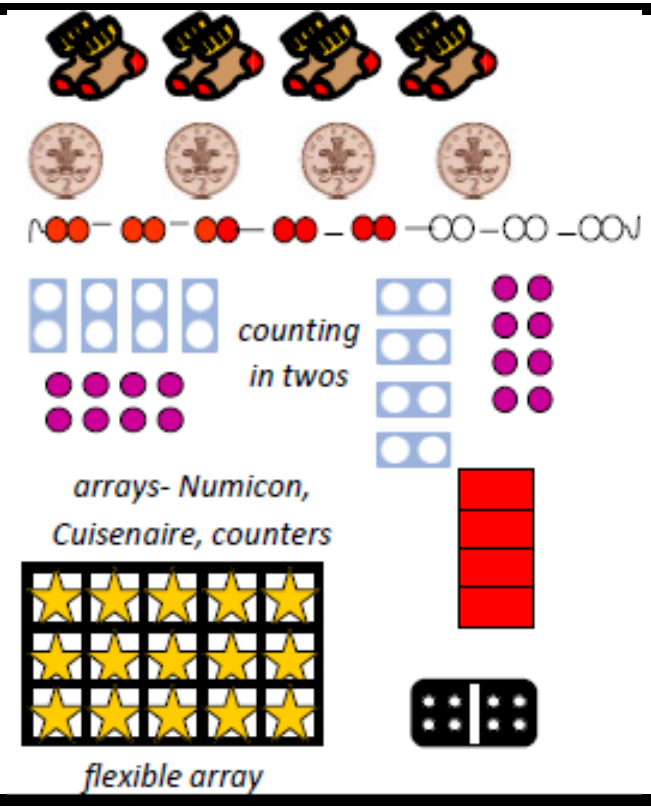
They solve problems, including doubling and halving

Year 1

Solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher

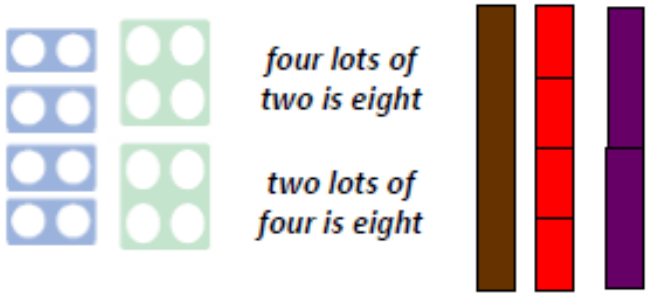
Count in multiples of 2s, 5s and 10s

Possible Concrete and Visual Representations	**Children's Recording**

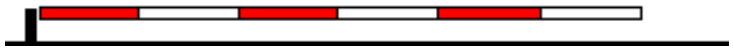


Practical only e.g. link to small world

Using concrete objects, pictorial representations and arrays with the support of an adult – take photographs/draw pictures – if using Numicon small icons could be stuck in



track with cuisenaire



Multiplication

Pupils develop the concept of multiplication and division and are enabled to use these operations flexibly. Multiplication and division should be taught together.

End of Year Expectations

Year 2

Recall and use multiplication facts for the 2,5,and 10 multiplication tables, relate to grouping, including recognising odd and even numbers

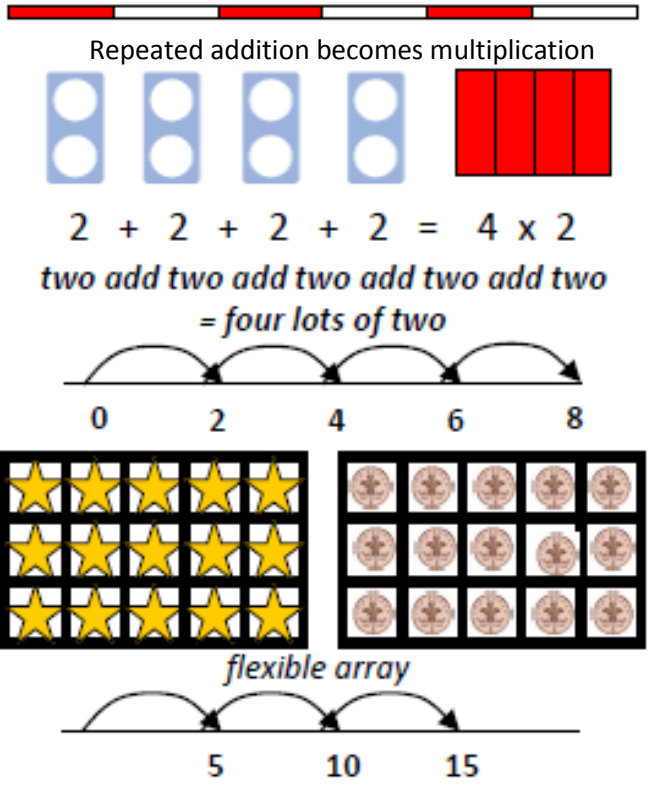
Calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication and equals signs

Show that multiplication of two numbers can be done in any order (commutative)

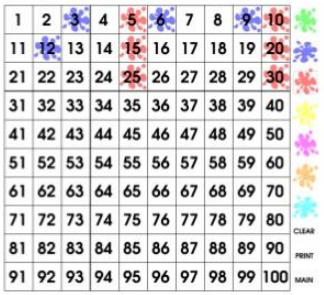
Solve problems involving multiplication, using materials, arrays, repeated addition, mental methods, and multiplication facts, including problems in context

Count in steps of 2, 3,and 5 from 0 and in tens from any number, forwards and backwards

Possible Concerete and Visual Representations

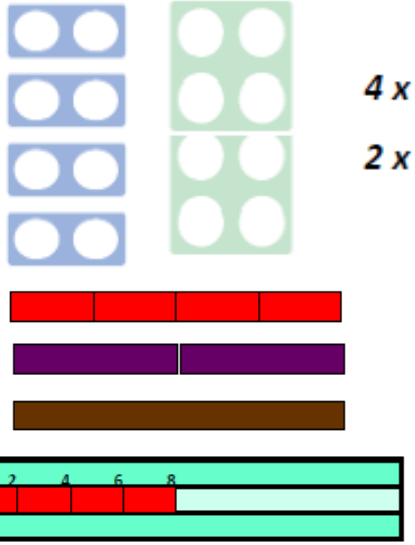


Hundred square/Splat square



Teacher Modelling/Children's Recording

Record practical work as number sentences



Commutative

Partitioning for larger numbers

11 x 3

10 x 3 = 30

1 x 3 =3

30 + 3 = 33

Multiplication

Pupils develop the concept of multiplication and division and are enabled to use these operations flexibly. Multiplication and division should be taught together.

End of Year Expectations

Year 3

Count from 0 in multiples of 4, 8, 50 and 100;

Recall and use multiplication facts for the 3, 4 and 8 multiplication tables

Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods

Solve problems, including missing number problems, involving multiplication, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects

Year 4

Count in multiples of 6, 7, 9, 25 and 1000

Recall multiplication and division facts for multiplication tables up to 12 × 12

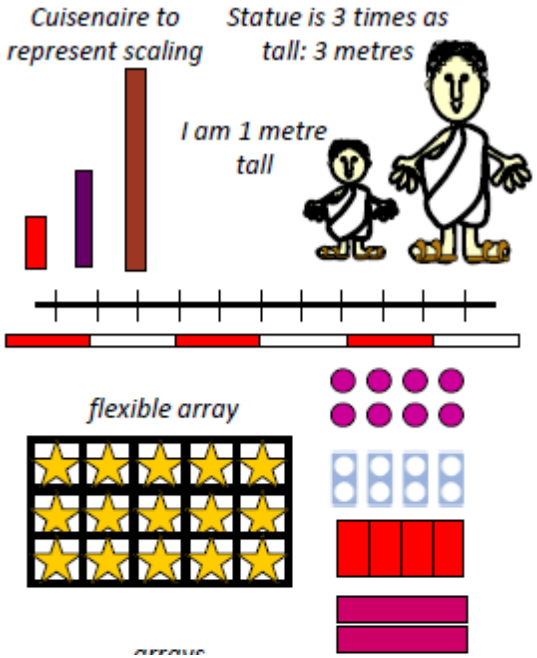
Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers

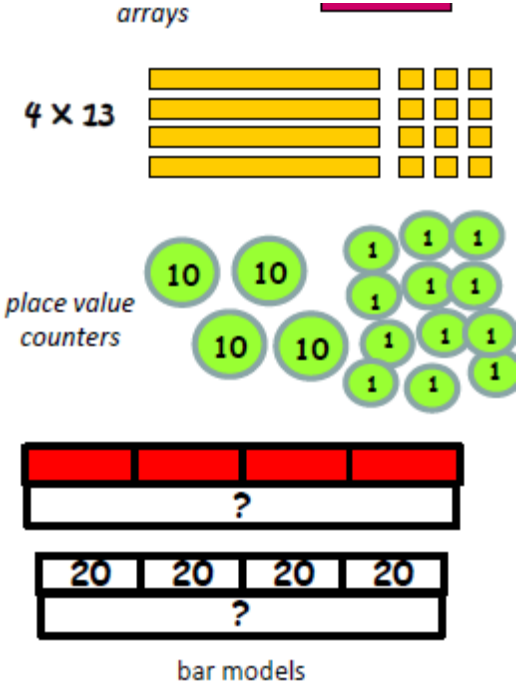
Recognise and use factor pairs and commutativity in mental calculations

Multiply two-digit and three-digit numbers by a one-digit number using formal written layout

Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects

Possible Concrete and Visual Representations





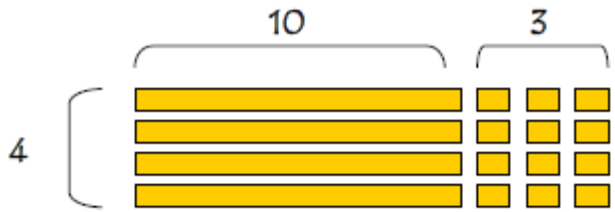
Multiplication	Pupils develop the concept of multiplication and division and are enabled to use these operations flexibly. Multiplication and division should be taught together.

Teacher Modelling/Children's Recording

Year 3 to use the grid method as their primary method of written multiplication. Year 4 to revise grid method and move on to long/short multiplication.

Children must use manipulatives alongside algorithms

4 x 13 'four lots of thirteen'



Grid method

x	10	3
4	40	12

40 + 12 = 52

x	100	30	3
4	400	120	12

400 + 120 + 12 = 532

Long multiplication two or three digit by one digit (begin by showing alongside grid method)

T	U
1	3
X	4
1	2
4	0
5	2

H	T	U
1	3	3
x		4
	1	2
1	2	0
4	0	0
5	3	2

Progressing to developing fluency in short multiplication

T	U
1	3
x	4
5	2
1	

H	T	U
1	3	3
	x	4
5	3	2
1	1	

Start with digits that are below five so children can practise method without encountering difficulty with multiplication tables

Multiplication	**Pupils develop the concept of multiplication and division and are enabled to use these operations flexibly. Multiplication and division should be taught together.**

End of Year Expectations

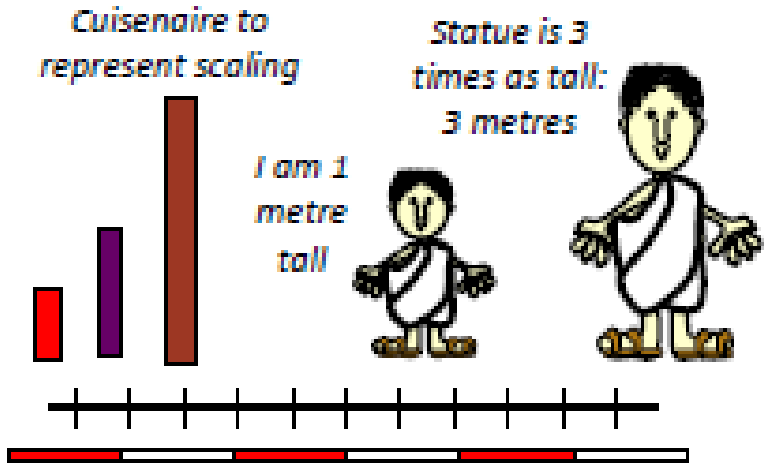
Year 5

- Multiply numbers mentally drawing upon known facts
- Multiply whole numbers and those involving decimals by 10, 100 and 1000
- Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- Solve problems involving multiplication

Year 6

- Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- Perform mental calculations, including with mixed operations and large numbers
- Use their knowledge of the order of operations to carry out calculations involving the four operations
- Solve problems involving addition, subtraction, multiplication and division

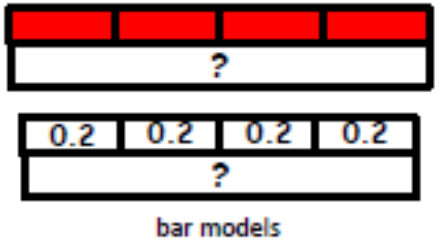
Possible Concrete and Visual Representations



arrays

place value counters

flexible array



bar models

Multiplication	**Pupils develop the concept of multiplication and division and are enabled to use these operations flexibly. Multiplication and division should be taught together.**

Teacher Modelling/Children's Recording

Children might use manipulatives alongside algorithms

X	10	8
7	70	56

```
  1  8
X    7
-------
  5  6
+ 7  0
-------
1  2  6
```

X	10	8
10	100	80
3	30	24

```
  1  8
X 1  3
-------
  5  4
+ 1 8  0
-------
2  3  4
```

Short multiplication (3 or 4 digits by 1 digit)

```
Th  H  T  U
1  3  2  4
X        6
-----------
7  9  4  4
1  1  2
```

```
   U   1/10  1/100
   3 . 2  4
x  6 .
-----------
  19.4  4
   1   2
```

Long multiplication

```
   Th   H   T   U
    1   3   2   4
X           2   6
-----------------
    7   9   4   4
    1   1   2

+   2   6   4   8   0
-----------------
    3   4   4   2   4
1   1   1
```

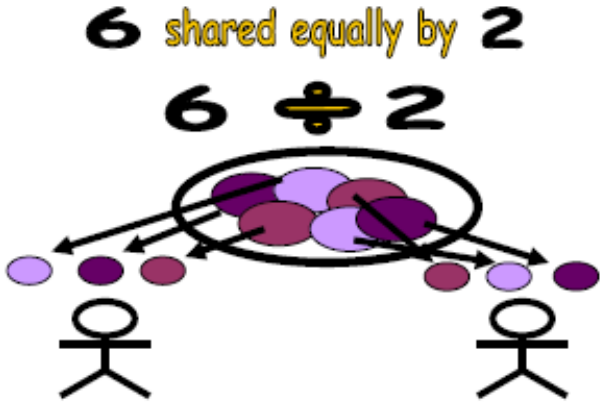
```
    T   U   1/10  1/100
        3 . 2   4
x       2 6 .
-------------
    1 9 . 4   4
        1   2

+   6 4 . 8   0
-------------
    8 4 . 2   4
```

Structures for Division (Haylock and Cockburn 2008)

Children should experience problems with the different division structures in a range of practical and relevant contexts e.g. money and measurement

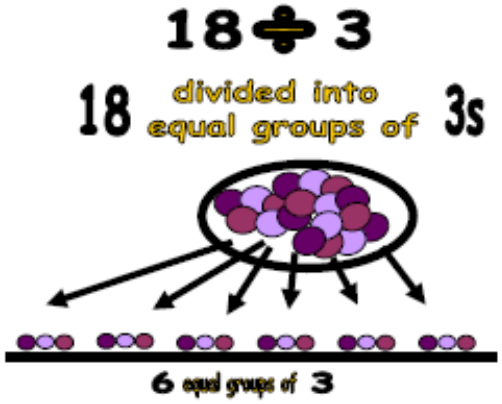
Equal-sharing

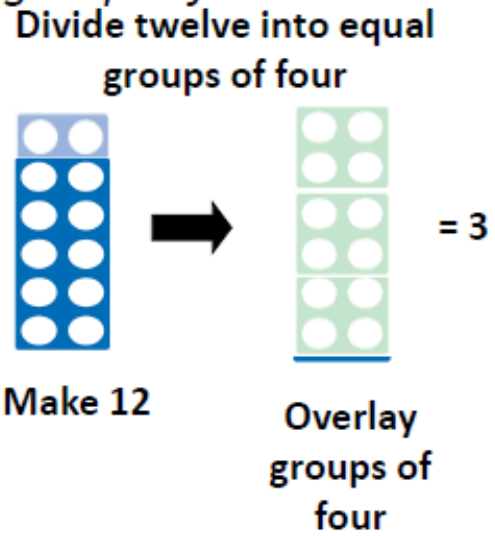
Sharing equally between
How many (much) each?



Inverse of multiplication (Grouping)

So many lots (sets/groups) of so many
Share equally in to groups of …





Ratio structure

comparison
inverse of scaling structure of multiplication
scale factor (decrease)

Barney earns three times more than Fred. If Barney earns £900 how much does Fred earn?

Jo's journey to school is three times as long as Ella's. If Jo walks to school in 30 minutes how long does it take Ella?

Division	Pupils develop the concepts of multiplication and division and are enabled to use these operations flexibly. Multiplication and division should be taught together.

End of Year Expectations

Reception

- They solve problems, including halving and sharing

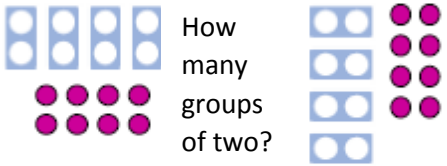
Year 1

- **Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher**

Possible Concrete and Visual Representations	Children's Recording

How many groups of two?

straw bundles



Numicon and counter arrays

four lots of two *two lots of four* *Cuisenaire*

doubling *flexible array*

bar models

Practical only e.g. link to small world

Using concrete objects, pictorial representations and arrays with the support of an adult – take photographs/draw pictures – if using Numicon small icons could be stuck in

Eight can be divided into four equal groups of two or two equal groups of four

Division	Pupils develop the concepts of multiplication and division and are enabled to use these operations flexibly. Multiplication and division should be taught together.

End of Year Expectations

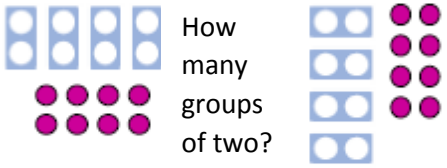
Year 2

- Calculate mathematical statements for division within the multiplication tables and write them using the division and equals signs
- Recall and use multiplication and division facts for the 2,5,and 10 multiplication tables, relate to grouping and sharing
- Solve problems involving division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in context

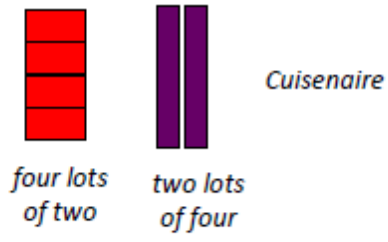
Possible Concrete and Visual Representations	Children's recording

How many groups of two?

straw bundles

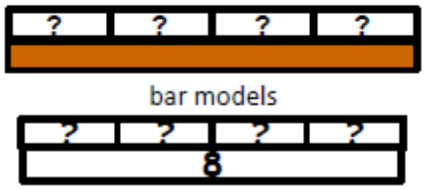


Numicon and counter arrays



four lots of two *two lots of four* *Cuisenaire*

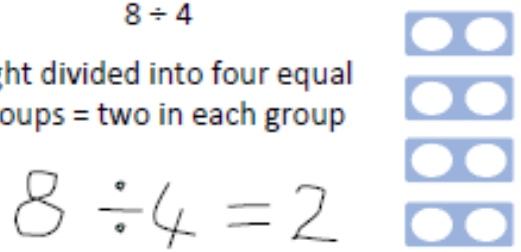
doubling *flexible array*

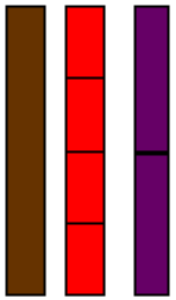


Record as number sentences using ÷ and =

8 ÷ 4

Eight divided into four equal groups = two in each group





Eight can be divided into four equal groups of two or two equal groups of four

Division	Pupils develop the concepts of multiplication and division and are enabled to use these operations flexibly. Multiplication and division should be taught together.

End of Year Expectations

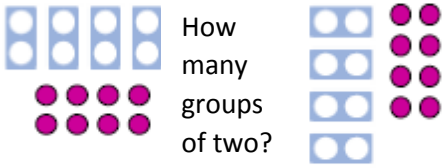
Year 2

- Calculate mathematical statements for division within the multiplication tables and write them using the division and equals signs
- Recall and use multiplication and division facts for the 2,5,and 10 multiplication tables, relate to grouping and sharing
- Solve problems involving division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in context

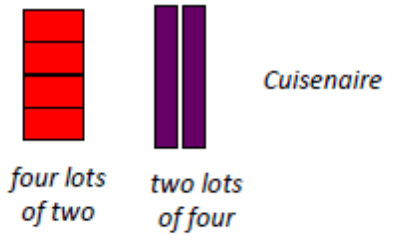
Possible Concrete and Visual Representations	Children's Recording

How many groups of two?

straw bundles

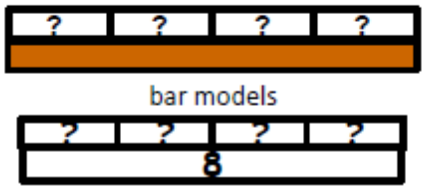


Numicon and counter arrays



four lots of two *two lots of four* *Cuisenaire*

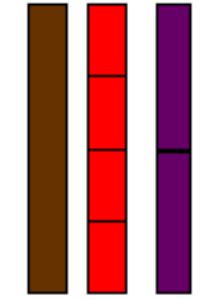
doubling *flexible array*



Record as number sentences using ÷ and =

8 ÷ 4

Eight divided into four equal groups = two in each group



8 ÷ 4 = 2

Eight can be divided into four equal groups of two or two equal groups of four

Division	Pupils develop the concepts of multiplication and division and are enabled to use these operations flexibly. Multiplication and division should be taught together.

End of Year Expectations

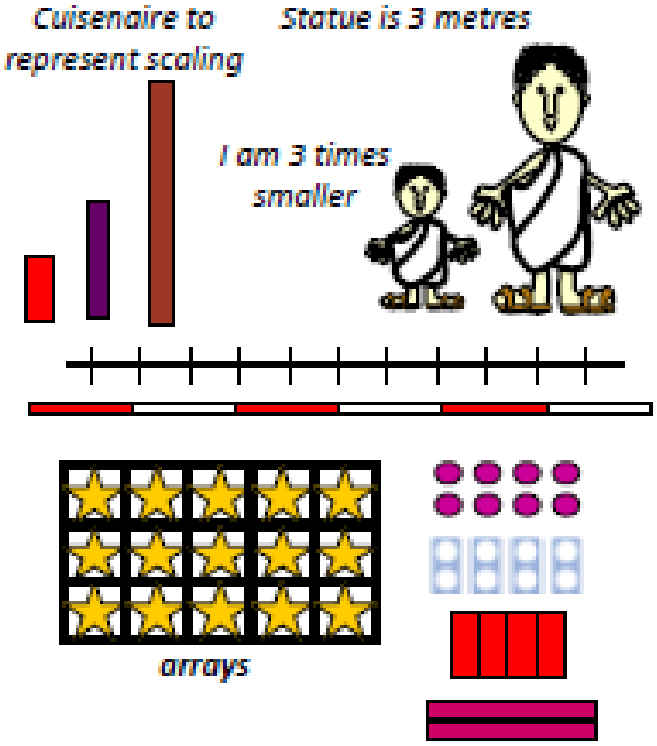
Year 3

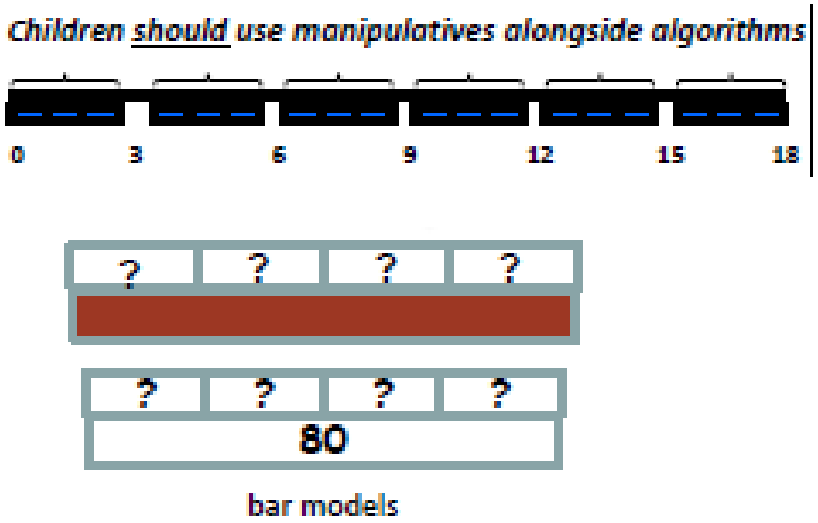
- Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- Write and calculate mathematical statements for division using the multiplication tables that they know
- Solve problems, including missing number problems, involving division

Year 4

- Recall multiplication and division facts for multiplication tables up to 12 × 12
- Use place value, known and derived facts to divide mentally, including: dividing by 1

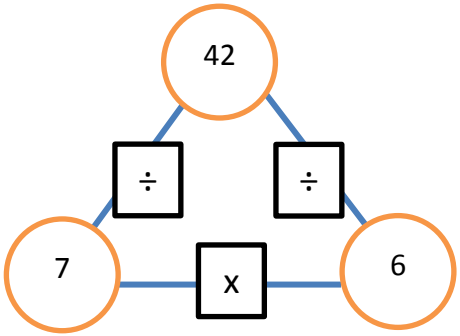
Possible Concrete and Visual Representations





bar models

Children's Recording



7 x 6 = 42

6 x 7 = 42

42 ÷ 6 = 7

42 ÷ 7 = 6

Division	Pupils develop the concepts of multiplication and division and are enabled to use these operations flexibly. Multiplication and division should be taught together.

End of Year Expectations

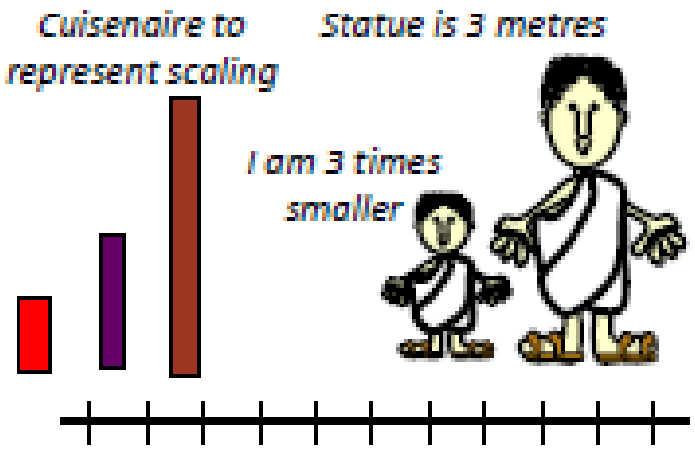
Year 5

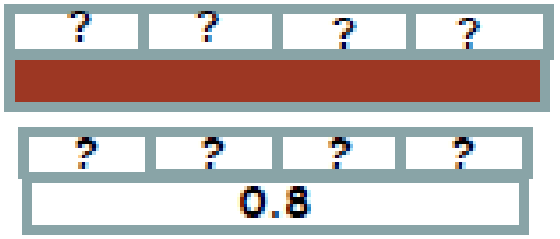
- Divide numbers mentally drawing upon known facts
- Divide whole numbers and those involving decimals by 10, 100 and 1000
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign

Year 6

- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- Solve problems involving addition, subtraction, multiplication and division

Possible Concrete and Visual Representations

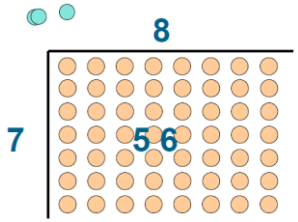


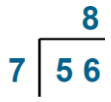


bar models

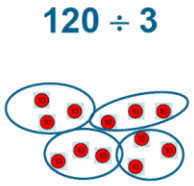

flexible arrays

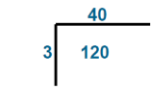
The array is an image for division too





The power of the place value: counters for larger numbers





Multiplication

Pupils develop the concept of multiplication and division and are enabled to use these operations flexibly. Multiplication and division should be taught together.

Teacher Modelling/Children's Recording

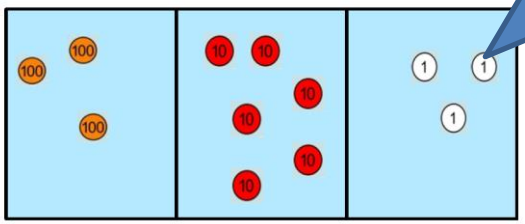
Year 5

Manipulatives *must* be used alongside algorithums

Short division using place value counters

363 ÷ 3 =

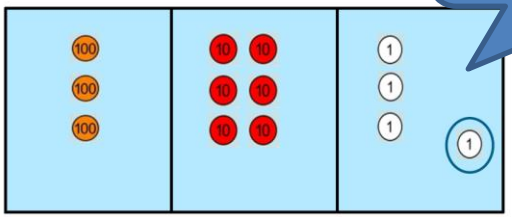
$$\begin{array}{r} 1\,2\,1 \\ 3\overline{)\,3\,6\,3} \end{array}$$



How many 3 hundreds in 300? How many 30s in 60? How many 3s in 3?

364 ÷ 3 =

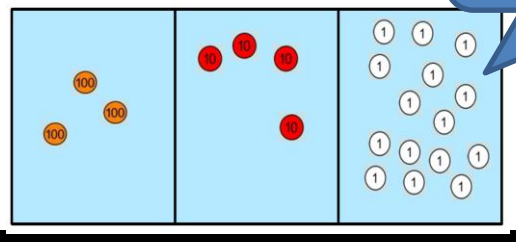
$$\begin{array}{r} 1\,2\,1 \text{ rem } 1 \\ 3\overline{)\,3\,6\,4} \end{array}$$



How many 3 hundreds in 300? How many 30s in 60? How many 3s in 4?

345 ÷ 3 =

$$\begin{array}{r} 1\,1\,5 \\ 3\overline{)\,3\,4^{1}5} \end{array}$$



How many 3 hundreds in 300? How many 30s in 40? How many 3s in 15?

Year 6

Children might use manipulatives alongside algorithms

without remainder

560 ÷ 4

$$\begin{array}{r} 1\,4\,0 \\ 4\overline{)\,5^{1}6\,0} \end{array}$$

short division

with remainder **561 ÷ 4**

$$\begin{array}{r} 1\,4\,0\ r\,1 \\ 4\overline{)\,5^{1}6\,1} \end{array}$$

remainder as a fraction

$$\begin{array}{r} 1\,1\,2\ \ 2/5 \\ 5\overline{)\,5\,6^{1}4} \end{array}$$

remainder as a decimal

564 ÷ 5

$$\begin{array}{r} 1\,1\,2.8 \\ 5\overline{)\,5\,6^{1}4^{4}.0} \end{array}$$

long division

560 ÷ 24

$$\begin{array}{r} 2\,3\ r\,8 \\ 24\overline{)\,5\,6\,0} \\ -4\,8 \\ \hline 8\,0 \\ -7\,2 \\ \hline 8 \end{array}$$

remainder as a whole number

$$\begin{array}{r} 2\,3\ \ 8/24\ (1/3) \\ 24\overline{)\,5\,6\,0} \\ -4\,8 \\ \hline 8\,0 \\ 7\,2 \\ \hline 8 \end{array}$$

remainder as a fraction in its lowest form

$$\begin{array}{r} 2\,3.3 \\ 24\overline{)\,5\,6\,0.0} \\ -4\,8 \\ \hline 8\,0 \\ 7\,2 \\ \hline 8\,0 \\ 7\,2 \end{array}$$

remainder as a decimal